

# TOPICS IN HIDDEN SYMMETRIES. VI.

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ABSTRACT. In this note devoted to some aspects of the inverse problem of representation theory the attention is concentrated on the interrelations between various algebraic structures (algebras with operators) unraveled by different solutions of the same inverse problem. The Lie algebras and Jordan triple systems with operators are considered.

This article being a continuation of five previous parts [1] as illustrates the general ideology presented in the review [2] as explicates its new features. The attention is now concentrated presumably on the interrelations between various algebraic structures (algebras with operators) unraveled by different solutions of the same inverse problem of the representation theory.

## 1. TOPIC ELEVEN: JORDAN DESIGNS ON LIE $\tilde{m}$ YB-ALGEBRAS

### 1.1. Lie $\tilde{m}$ YB-algebras and Jordan designs.

**Definition 1A** [3]. The Lie  $\tilde{m}$ YB-algebra is a pair  $(\mathfrak{g}, R)$ , where  $\mathfrak{g}$  is the Lie algebra with brackets  $[\cdot, \cdot]$  and  $R$  is an operator in it such that

$$R[RX, Y] + R[X, RY] = [RX, RY] + R^2[X, Y].$$

The concept of the Lie  $\tilde{m}$ YB-algebra was motivated by the constructions of the paper [4], the main identity of the Lie  $\tilde{m}$ YB-algebra coincides with the modified classical Yang-Baxter equation if  $R^2 = 1$  (such condition often holds [4]). Examples of Lie  $\tilde{m}$ YB-algebras were considered in [3].

*Remark 1* [3]. In any Lie  $\tilde{m}$ YB-algebra  $(\mathfrak{g}, R)$  the bracket

$$[X, Y]_R = [RX, Y] + [X, RY] - R[X, Y]$$

supplies the linear space  $\mathfrak{g}$  by a structure of the Lie algebra.

*Remark 2* [3]. Let  $(\mathfrak{g}, R)$  be a Lie  $\tilde{m}$ YB-algebra and  $f(x)$  be a polynomial then  $(\mathfrak{g}, f(R))$  is the Lie  $\tilde{m}$ YB-algebra, in particular,  $(\mathfrak{g}, 1 + \lambda R)$  is a Lie  $\tilde{m}$ YB-algebra for any  $\lambda$ . Hence, the (compatible) Lie brackets  $[\cdot, \cdot]_{1+\lambda R}$  supply the linear space  $\mathfrak{g}$  by a structure of the linear bunch of Lie algebras.

**Definition 1B** [5]. The *Jordan design* on the Lie algebra  $\mathfrak{g}$  with the commutator  $[\cdot, \cdot]$  is a  $\mathfrak{g}$ -equivariant structure of a Jordan triple system on  $\mathfrak{g}$  with the trilinear operation  $\langle \cdot, \cdot, \cdot \rangle$  such that

$$[A, \langle X, A, X \rangle] + [X, \langle A, X, A \rangle] = 0$$

for any  $X$  and  $Y$  from  $\mathfrak{g}$ .

Remind that the *Jordan triple system* [6] is the linear space  $V$  supplied by a trilinear operation  $\langle \cdot, \cdot, \cdot \rangle$ , which obeys the following identity

$$\langle x, \langle a, z, b \rangle, y \rangle = \langle \langle x, a, y \rangle, b, z \rangle + \langle \langle y, a, z \rangle, b, x \rangle - \langle \langle x, b, y \rangle, a, z \rangle.$$

Examples of Jordan designs on Lie algebras were considered in [5]. Note that both additional structures on Lie algebras (the structure of Lie  $\tilde{\mathfrak{m}}$ YB-algebra and the structure of Jordan design) naturally appear in the context of the inverse problem of representation theory. So it is reasonable to discuss their interrelations and to consider Jordan designs on  $\tilde{\mathfrak{m}}$ YB-Lie algebras now. It will be demonstrated how some nontrivial conditions of the structural compatibility appear.

Let  $(\mathfrak{g}, R)$  be a Lie  $\tilde{\mathfrak{m}}$ YB-algebra with the Jordan design  $\langle \cdot, \cdot, \cdot \rangle$ . Define a new triple product as

$$\begin{aligned} \langle X, Y, Z \rangle_R &= \langle X, RY, RZ \rangle + \langle RX, Y, RZ \rangle + \langle RX, RY, Z \rangle \\ &\quad - R \langle RX, Y, Z \rangle - R \langle X, RY, Z \rangle - R \langle X, Y, RZ \rangle + R^2 \langle X, Y, Z \rangle. \end{aligned}$$

Note that  $\langle \cdot, \cdot, \cdot \rangle$  depends on  $R$  quadratically whereas  $[\cdot, \cdot]_R$  depends on  $R$  linearly. The construction of this new product can be naturally generalized to the higher  $n$ -ary operations.

**Definition 1C.** The *Jordan triple  $\tilde{\mathfrak{m}}$ YB-system* is a pair  $(\mathfrak{g}, R)$ , where  $\mathfrak{g}$  is the Jordan triple system with triple product  $\langle \cdot, \cdot, \cdot \rangle$  and  $R$  is an operator in it such that

$$R \langle RX, Y, Z \rangle + R \langle X, Y, RZ \rangle = \langle RX, Y, RZ \rangle + R^2 \langle X, Y, Z \rangle.$$

*Remark 3.* The operation  $\langle \cdot, \cdot, \cdot \rangle_R$  has the reduced form

$$\langle X, Y, Z \rangle_R = \langle RX, RY, Z \rangle + \langle X, RY, RZ \rangle - R \langle X, RY, Z \rangle$$

in any Jordan triple  $\tilde{\mathfrak{m}}$ YB-system.

**Theorem 1A.** In any Jordan triple  $\tilde{\mathfrak{m}}$ YB-system  $(\mathfrak{g}, R)$  the bracket  $\langle \cdot, \cdot, \cdot \rangle_R$  supplies the linear space  $\mathfrak{g}$  by a new structure of the Jordan triple system.

*Remark 4.* The equality

$$R \langle X, Y, Z \rangle_R = \langle RX, RY, RZ \rangle$$

holds in any Jordan triple  $\tilde{\mathfrak{m}}$ YB-system.

Let us now consider the Lie  $\tilde{\mathfrak{m}}$ YB-algebra  $(\mathfrak{g}, R)$  supplied by the  $\mathfrak{g}$ -equivariant Jordan trilinear operation  $\langle \cdot, \cdot, \cdot \rangle$  such that the pair  $(\mathfrak{g}, R)$  is a Jordan triple  $\tilde{\mathfrak{m}}$ YB-system. The following proposition holds.

**Proposition 1.** Let  $(\mathfrak{g}, R)$  be a Lie  $\tilde{\text{mYB}}$ -algebra and  $\langle \cdot, \cdot, \cdot \rangle$  be a  $\mathfrak{g}$ -equivariant Jordan trilinear operation on  $\mathfrak{g}$  such that  $(\mathfrak{g}, R)$  is a Jordan  $\tilde{\text{mYB}}$ -triple system. The Jordan trilinear operation  $\langle \cdot, \cdot, \cdot \rangle_R$  is  $\mathfrak{g}_R$ -equivariant, where  $\mathfrak{g}_R$  is the Lie algebra  $\mathfrak{g}$  supplied by the bracket  $[\cdot, \cdot]_R$ .

**Definition 1D.** The Jordan  $\tilde{\text{mYB}}$ -design on the Lie  $\tilde{\text{mYB}}$ -algebra  $(\mathfrak{g}, R)$  is a Jordan design on the Lie algebra  $\mathfrak{g}$  such that  $(\mathfrak{g}, R)$  is a Jordan triple  $\tilde{\text{mYB}}$ -system.

Let us now formulate the main theorem on Jordan  $\tilde{\text{mYB}}$ -designs.

**Theorem 1B.** If  $\langle \cdot, \cdot, \cdot \rangle$  is a Jordan  $\tilde{\text{mYB}}$ -design on the Lie  $\tilde{\text{mYB}}$ -algebra  $(\mathfrak{g}, R)$  then the bracket  $\langle \cdot, \cdot, \cdot \rangle_R$  realizes a Jordan design on the Lie algebra  $\mathfrak{g}_R$ .

*Example 1.* Let  $\mathfrak{g}$  be the Lie algebra  $\mathfrak{so}(3, \mathbb{C})$  of all skew-symmetric  $n \times n$  matrices and  $\langle \cdot, \cdot \rangle$  be the natural bilinear symmetric form on it. Put  $RX = \langle X_0, X \rangle$  and  $\langle X, Y, Z \rangle = \langle X, Y \rangle Z + \langle Y, Z \rangle X$  then  $(\mathfrak{g}, R)$  is a Lie  $\tilde{\text{mYB}}$ -algebra with a Jordan  $\tilde{\text{mYB}}$ -design.

Jordan  $\tilde{\text{mYB}}$ -designs on Lie  $\tilde{\text{mYB}}$ -algebras may be used for a description of nonhamiltonian interactions governed by Jordan designs [5] in external (magnetic-type) fields.

**1.2. Lie bi- $\tilde{\text{mYB}}$ -algebras and Jordan designs.** Let us now discuss the Jordan designs on Lie bi- $\tilde{\text{mYB}}$ -algebras.

**Definition 2A.** The Lie bi- $\tilde{\text{mYB}}$ -algebra is a Lie algebra  $\mathfrak{g}$  with bracket  $[\cdot, \cdot]$  supplied by two commuting operators  $R_1$  and  $R_2$  such that  $(\mathfrak{g}, R_1)$  and  $(\mathfrak{g}, R_2)$  are the Lie  $\tilde{\text{mYB}}$ -algebras with identical brackets  $[\cdot, \cdot]_{R_1}$  and  $[\cdot, \cdot]_{R_2}$ . A Lie bi- $\tilde{\text{mYB}}$ -algebra  $(\mathfrak{g}, R_1, R_2)$  is called *even-tempered* if the identities

$$\begin{aligned} [R_1 X, R_2 Y] + [R_2 X, R_1 Y] - R_1 R_2 [X, Y] &= [R_1^2 X, Y] + [X, R_1^2 Y] - R_1^2 [X, Y], \\ [R_1 X, R_2 Y] + [R_2 X, R_1 Y] - R_1 R_2 [X, Y] &= [R_2^2 X, Y] + [X, R_2^2 Y] - R_2^2 [X, Y] \end{aligned}$$

hold.

Examples of (even-tempered) bi- $\tilde{\text{mYB}}$ -algebras were considered in [3]. The main example is exposed below.

*Example 2.* Let  $\mathfrak{A}$  be an associative algebra then its commutator algebra  $\mathfrak{A}_{[\cdot, \cdot]}$  is a Lie bi- $\tilde{\text{mYB}}$ -algebra, where  $R_1(X) = R_Q^r(X) = XQ$  or  $R_2(X) = R_Q^l(X) = QX$  are the operators of the left or right multiplication on the elements  $Q$  of the associative algebra  $\mathfrak{A}$ . Moreover,

$$[X, Y]_{R_1} = [X, Y]_{R_2} = XQY - YQX.$$

The Lie  $\tilde{\text{mYB}}$ -algebra  $(\mathfrak{g}, R)$  is a Lie bi- $\tilde{\text{mYB}}$ -algebra with  $R_1 = R_2 = R$ . Such Lie bi- $\tilde{\text{mYB}}$ -algebra is not even-tempered in general.

*Remark 5* [3]. A Lie  $\tilde{\text{mYB}}$ -algebra  $(\mathfrak{g}, R)$  is the Lie bi- $\tilde{\text{mYB}}$ -algebra if and only if there exists a derivative  $\xi$  of the Lie algebra  $\mathfrak{g}$  commuting with  $R$  such that

$$[\xi X, \xi Y] = [SX, Y] + [X, SY] - S[X, Y], \quad S = R\xi,$$

and  $R_1 = R, R_2 = R + \xi$ .

Note that the operator  $\xi$  is a derivative of both brackets  $[\cdot, \cdot]$  and  $[\cdot, \cdot]_R$ .

*Remark 6* [3]. For any Lie bi- $\tilde{m}$ YB-algebra  $(\mathfrak{g}, R_1, R_2)$  and any polynomial  $f(x)$  the triple  $(\mathfrak{g}, f(R_1), f(R_2))$  is a Lie bi- $\tilde{m}$ YB-algebra.

*Remark 7* [3]. The identities in the even-tempered Lie bi- $\tilde{m}$ YB-algebra may be rewritten in terms of  $R$  and  $\xi$  as

$$[RX, \xi Y] + [\xi X, RY] - R\xi[X, Y] = [R^2X, Y] - 2[RX, RY] + [X, R^2Y].$$

Note that if one puts  $R_0 = \frac{1}{2}(R_1 + R_2)$  then the bracket  $[\cdot, \cdot]_{R_0}$  defined as  $[X, Y]_{R_0} = [R_0X, Y] + [X, R_0Y] - R_0[X, Y]$  coincides with brackets  $[\cdot, \cdot]_{R_i}$  ( $i = 1, 2$ ), however, the pair  $(\mathfrak{g}, R_0)$  does not constitute a Lie bi- $\tilde{m}$ YB-algebra.

Let us now define Jordan triple  $\tilde{m}$ YB-triple systems.

**Definition 2B.** The *Jordan triple bi- $\tilde{m}$ YB-system* is a Jordan triple system  $\mathfrak{g}$  with trilinear operation  $\langle \cdot, \cdot, \cdot \rangle$  supplied by two commuting operators  $R_1$  and  $R_2$  such that  $(\mathfrak{g}, R_1)$  and  $(\mathfrak{g}, R_2)$  are the Jordan triple  $\tilde{m}$ YB-systems with identical trilinear operations  $\langle \cdot, \cdot, \cdot \rangle_{R_1}$  and  $\langle \cdot, \cdot, \cdot \rangle_{R_2}$ . A Jordan triple bi- $\tilde{m}$ YB-system  $(\mathfrak{g}, R_1, R_2)$  is called *normal* and *even-tempered* if the identities

$$\begin{aligned} \langle X, R_1R_2Y, Z \rangle &= \langle R_1X, Y, R_2Z \rangle + \langle R_2X, Y, R_1Z \rangle - R_1R_2 \langle X, Y, Z \rangle \\ &= \langle R_1X, R_1Y, Z \rangle + \langle X, R_1Y, R_1Z \rangle - R_1 \langle X, R_1Y, Z \rangle \\ &= \langle R_2X, R_2Y, Z \rangle + \langle X, R_2Y, R_2Z \rangle - R_2 \langle X, R_2Y, Z \rangle \end{aligned}$$

and

$$\begin{aligned} \langle R_1X, R_1R_2Y, R_2Z \rangle &+ \langle R_2X, R_1R_2Y, R_2Z \rangle - R_1R_2 \langle X, R_1R_2Y, Z \rangle \\ &= \langle R_1^2X, R_1^2Y, Z \rangle + \langle X, R_1^2Y, R_1^2Z \rangle - R_1^2 \langle X, R_1^2Y, Z \rangle \\ &= \langle R_2^2X, R_2^2Y, Z \rangle + \langle X, R_2^2Y, R_2^2Z \rangle - R_2^2 \langle X, R_2^2Y, Z \rangle, \end{aligned}$$

respectively, hold.

*Remark 8.* The defining identities for normal Jordan triple  $\tilde{m}$ YB-systems may be concisely rewritten as

$$\langle X, Y, Z \rangle_R = \langle X, \varrho Y, Z \rangle,$$

where  $\varrho = R_1R_2$ .

*Exercise.* Rewrite the definition of Jordan triple  $\tilde{m}$ YB-systems and its proper subclasses of normal and even-tempered systems in terms of operators  $R = R_1$  and  $\xi = R_2 - R_1$ .

Note that any Jordan triple  $\tilde{m}$ YB-system  $(\mathfrak{g}, R)$  is a Jordan triple bi- $\tilde{m}$ YB-system with  $R_1 = R_2 = R$ . Such Jordan triple bi- $\tilde{m}$ YB-system is neither normal nor even-tempered in general.

*Example 3.* Let  $\mathfrak{A}$  be an associative algebra then the associated Jordan triple system  $\mathfrak{A}_{\langle \cdot, \cdot, \cdot \rangle}$  (where  $\langle X, Y, Z \rangle = XYZ + ZYX$ ) is a Jordan triple bi- $\tilde{m}$ YB-system, where  $R_1(X) = R_Q^r(X) = XQ$  or  $R_2(X) = R_Q^l(X) = QX$  are the operators of the left or right multiplication on the elements  $Q$  of the associative algebra  $\mathfrak{A}$ . Moreover,

$$\langle X, Y, Z \rangle_{R_1} = \langle X, Y, Z \rangle_{R_2} = XQYQZ - ZQYQX.$$

**Proposition 2.** *The Jordan triple bi- $\tilde{m}$ YB-system of example 3 is normal and even-tempered.*

Analizing the example 3 one may write a huge number of new identities besides ones for normal and even-tempered Jordan triple  $\tilde{m}$ YB-systems, which hold for systems of this example, the most interesting of which is the  $\varrho$ -identity

$$\langle \varrho X, Y, \varrho Z \rangle = \varrho \langle X, \varrho Y, Z \rangle.$$

For the triple systems of example 3 the mapping  $\varrho : X \mapsto \varrho X$  is well-known quadratic mapping  $X \mapsto QXQ = \frac{1}{2} \langle Q, X, Q \rangle$  [7].

*Remark 9.* In view of the remark 8  $\varrho$ -identity may be concisely rewritten as

$$\varrho \langle X, Y, Z \rangle_{\varrho} = \langle \varrho X, Y, \varrho Z \rangle,$$

where  $\langle \cdot, \cdot, \cdot \rangle_{\varrho}$  is the renotation for the coinciding brackets  $\langle \cdot, \cdot, \cdot \rangle_{R_i}$  ( $i = 1, 2$ ).

**Definition 2C.** The *Jordan bi- $\tilde{m}$ YB-design* on the Lie bi- $\tilde{m}$ YB-algebra  $(\mathfrak{g}, R_1, R_2)$  is a Jordan design on the Lie algebra  $\mathfrak{g}$  such that  $(\mathfrak{g}, R_1, R_2)$  is a Jordan triple bi- $\tilde{m}$ YB-system.

The design will be called normal or even-tempered if the Jordan triple  $\tilde{m}$ YB-system is normal or even-tempered.

**Proposition 3.** *The Jordan triple bi- $\tilde{m}$ YB-system of example 3 supplies the even-tempered Lie bi- $\tilde{m}$ YB-algebra of example 2 by a normal even-tempered Jordan bi- $\tilde{m}$ YB-design with  $\varrho$ -identity.*

## 2. TOPIC TWELVE: LIE $R\varrho$ -ALGEBRAS

**Definition 3.** The *Lie  $R\varrho$ -algebra* is a triple  $(\mathfrak{g}, R, \varrho)$ , where  $\mathfrak{g}$  is the Lie algebra with the bracket  $[\cdot, \cdot]$  and  $R, \varrho$  are two operators in it such that the following two identities

$$\begin{aligned} \varrho[X, Y]_{\varrho} &= [\varrho X, \varrho Y], \\ R[X, Y]_{\varrho} + \varrho[X, Y]_R &= [RX, \varrho Y] + [\varrho X, RY]. \end{aligned}$$

holds for all  $X$  and  $Y$  from  $\mathfrak{g}$ . Here

$$\begin{aligned} [X, Y]_R &= [RX, Y] + [X, RY] - R[X, Y], \\ [X, Y]_{\varrho} &= [\varrho X, Y] + [X, \varrho Y] - \varrho[X, Y] + [RX, RY] - R[X, Y]_R. \end{aligned}$$

A Lie  $R\varrho$ -algebra  $(\mathfrak{g}, R, \varrho)$  is called *regular* if the identity

$$R[X, Y]_R = 2([\varrho X, Y] + [X, \varrho Y])$$

holds.

**Proposition 4.** *Any even-tempered Lie bi- $\tilde{m}$ YB-algebra  $(\mathfrak{g}, R_1, R_2)$  is the regular Lie  $R\varrho$ -algebra with  $R = R_1 + R_2$ ,  $\varrho = R_1 R_2$ .*

The inverse statement is not true. Let us expose an example of the Lie  $R\varrho$ -algebra, which is not a Lie bi- $\tilde{m}$ YB-algebra.

*Example 4.* Let  $\mathfrak{g}$  be the Lie algebra  $\mathfrak{so}(n, \mathbb{C})$  of all skew-symmetric  $n \times n$  matrices and  $Q$  be a symmetric  $n \times n$  matrix. If one puts  $RX = QX + XQ$  and  $\varrho X = QXQ$  then  $(\mathfrak{g}, R, \varrho)$  will be a Lie  $R\varrho$ -algebra, however, there are no operators  $R_1, R_2$  in  $\mathfrak{g}$  constituting a Lie bi- $\tilde{m}$ YB-algebra  $(\mathfrak{g}, R_1, R_2)$  such that  $R_1 + R_2 = R$ ,  $R_1 R_2 = \varrho$ , in general.

**Proposition 4.** *The bracket  $[\cdot, \cdot]_\varrho$  obeys the Jacobi identity in any Lie  $R\varrho$ -algebra  $(\mathfrak{g}, R, \varrho)$ .*

Let us now explicate the relation between Lie  $R\varrho$ -algebras and quadratic  $\Gamma$ -bunches of Lie algebras analogous to one between Lie  $\tilde{m}$ YB-algebras and linear  $\Gamma$ -bunches of Lie algebras [3].

**Definition 4** [3]. The *(one-parametric) bunch of Lie algebras* is a linear space  $\mathfrak{g}$  supplied by a family of Lie brackets  $[\cdot, \cdot]_\lambda$  ( $\lambda \in \mathbb{R}$ ). The Lie algebra from  $\mathfrak{g}$ , which is defined by the bracket  $[\cdot, \cdot]_\lambda$ , is denoted by  $\mathfrak{g}_\lambda$ . A bunch of Lie algebras  $\mathfrak{g}$  is called the  $\Gamma$ -*bunch* if there are defined homomorphisms  $R_\lambda$  from the Lie algebras  $\mathfrak{g}_\lambda$  into the Lie algebra  $\mathfrak{g}_0$ , otherwords, the equality

$$R_\lambda[X, Y]_\lambda = [R_\lambda X, R_\lambda Y],$$

where  $[\cdot, \cdot] = [\cdot, \cdot]_0$ , holds for all  $X$  and  $Y$  from  $\mathfrak{g}$ .

Below there will be considered only smooth bunches, i.e. the bunches, which Lie brackets form a smooth family, and smooth  $\Gamma$ -bunches, for which the family of homomorphisms  $R_\lambda$  is also smooth.

**Proposition 5** (cf.[3]). *Let  $\mathfrak{g}$  be a smooth  $\Gamma$ -bunch of Lie algebras. Define the tangent bracket  $[\cdot, \cdot]_R$  and quadratic bracket  $[\cdot, \cdot]_\varrho$  as*

$$[X, Y]_R = \left. \frac{d[X, Y]_\lambda}{d\lambda} \right|_{\lambda=0} \quad \text{and} \quad [X, Y]_\varrho = \frac{1}{2} \left. \frac{d^2[X, Y]_\lambda}{d\lambda^2} \right|_{\lambda=0}.$$

*Then*

$$\begin{aligned} [X, Y]_R &= [RX, Y] + [X, RY] - R[X, Y], \\ [X, Y]_\varrho &= [\varrho X, Y] + [X, \varrho Y] - \varrho[X, Y] + [RX, RY] - R[X, Y]_R, \end{aligned}$$

*where*

$$R = \left. \frac{dR_\lambda}{d\lambda} \right|_{\lambda=0} \quad \text{and} \quad \varrho = \left. \frac{1}{2} \frac{d^2 R_\lambda}{d\lambda^2} \right|_{\lambda=0}.$$

A bunch of Lie algebras  $\mathfrak{g}$  is called quadratic if the Lie brackets  $[\cdot, \cdot]_\lambda$  from a quadratic family, otherwords,  $[\cdot, \cdot]_\lambda = [\cdot, \cdot] + \lambda[\cdot, \cdot]_R + \lambda^2[\cdot, \cdot]_\varrho$ . A  $\Gamma$ -bunch of Lie algebras is called linear if not only Lie brackets but also the homomorphisms  $R_\lambda$  form a quadratic family, i.e.  $R_\lambda = 1 + \lambda R + \lambda^2 \varrho$ .

**Theorem 2.** *There is a natural one-to-one correspondence between quadratic  $\Gamma$ -bunches of Lie algebras and Lie  $R\varrho$ -algebras.*

*Question.* What does the regularity condition in Lie  $R\varrho$ -algebras mean in terms of quadratic  $\Gamma$ -bunches of Lie algebras?

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